SOLUTIONS

Math 113 (Calculus 2)
Exam 2

Feb 26 – March 2, 2010

Instructions:

1. Work on scratch paper will not be graded.
2. Should you have need for more space than is allotted to answer a question, use the back of the page the problem is on and indicate this fact.
3. Simplify your answers. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, $\tan^{-1}(1)$, etc. must be simplified for full credit.
4. Calculators are not allowed.

For Instructor use only.

#	Possible	Earned		#	Possible	Earned
M.C.	32			12	8	
9	12			13	8	
10	12			14	8	
11	12			15	8	
				Total	100	

Answers to MC: 1A 2D 3D 4E 5B 6A 7F 8D

Multiple Choice (32 points). Each problem is worth 4 points. Fill in the answer to each problem on your scantron. Make sure your name, section and instructor is on your scantron.

1. What is the formula for the arc length of the graph of the function $y = f(x)$, $a \leq x \leq b$.

   A. $\int_a^b \sqrt{1 + (f'(x))^2}\, dx$ B. $\int_a^b (1 + (f'(x))^2)\, dx$ C. $\int_a^b (dx^2 + dy^2)$ D. None of these.

   ANSWER A

2. Find the length of the curve $x = \dfrac{y^4}{8} + \dfrac{1}{4y^2}$, $1 \leq y \leq 2$.

   A. 2 B. $2\frac{1}{4}$ C. $1\frac{7}{8}$ D. $2\frac{1}{16}$ E. $1\frac{15}{16}$

   ANSWER D

3. Find the surface area if the curve $y = \sqrt{9 - x^2}$, $1 \leq x \leq 2$ is rotated about the x-axis.

   A. 3π B. 4π C. 5π D. 6π E. 8π

   ANSWER D

4. What is the hydrostatic force on an inverted isosceles triangle aquarium window with base 2 ft. and height 3 ft. whose top is 3 ft. below the surface of the water if the density of water is 62.5 lbs/ft^3?

   A. 250 lbs. B. 300 lbs.
   C. 400 lbs. D. 500 lbs.
   E. 750 lbs. F. 1000 lbs.

   ANSWER E

5. An isosceles trapezoid is the end of a water trough filled to the top with water. Find the hydrostatic force on the trapezoid to the nearest pound if the top base is 3 ft., the bottom base is 2 ft., and the height is 1 ft. The density of water is 62.5 lbs/ft^3.

   A. 70 lbs. B. 73 lbs.
   C. 77 lbs. D. 81 lbs.
   E. 85 lbs. F. 89 lbs.

   ANSWER B

6. Find the sum of the infinite geometric series $1 + \frac{1}{4} + \frac{1}{16} + \cdots$.

   A. $\frac{4}{3}$ B. 1.4 C. 1.5 D. 1.6 E. $\frac{7}{4}$

   ANSWER A

7. Find the x coordinate of the centroid of the following system consisting of a rectangle and a quarter circle.

   A. $-\dfrac{6}{8+\pi}$ B. $-\dfrac{7}{8+\pi}$

   C. $-\dfrac{13}{16+2\pi}$ D. $-\dfrac{15}{16+2\pi}$

   E. $-\dfrac{19}{24+3\pi}$ F. $-\dfrac{20}{24+3\pi}$

   ANSWER F

8. Use the integral definition of $\ln x$ from Appendix G and the midpoint rule with $n = 2$ to approximate $\ln 3$.

   A. $\dfrac{57}{60}$ B. $\dfrac{67}{60}$ C. $\dfrac{77}{60}$ D. $\dfrac{16}{15}$ E. $\dfrac{7}{6}$

   ANSWER D

Short Answer (36%). Fill in the blank with the appropriate answer. Each problem is worth 12 points. A correct answer gets full credit. You will need to show your work for partial credit.

9. (a) If $f'(x) > 0$ and $f''(x) < 0$ for $a \leq x \leq b$, Order L_n, R_n, M_n and T_n where L_n is the left endpoint approximation, R_n is the right endpoint approximation, M_n is the midpoint rule, and T_n is the trapezoidal rule each using n subdivisions.

$$\mathrm{L}_n < \mathrm{T}_n < \mathrm{M}_n < \mathrm{R}_n$$

(b) Circle the integrals that converge and put an X over the integrals that diverge.

A. $\displaystyle\int_0^1 \frac{dx}{x^3}$ B. $\displaystyle\int_1^\infty \frac{dx}{x^3}$ C. $\displaystyle\int_1^\infty \frac{3+\sin 2x}{x^2}dx$ D. $\displaystyle\int_1^\infty \frac{3+\sin 2x}{\sqrt{x}}dx$

ANSWER: A and D diverge. B and C converge.

(c) If $f(x)$ is a continuous function on the interval $0 \leq x \leq 2$ and $f(0) = 1\frac{1}{2}$, $f(\frac{1}{2}) = 1\frac{3}{4}$, $f(1) = 1\frac{1}{2}$, $f(1\frac{1}{2}) = 1\frac{1}{4}$, and $f(2) = 2\frac{1}{2}$, use Simpson's rule with $n = 4$ to estimate $\int_0^2 f(x)dx$.

SOLUTION:

$$S_4 = \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)\left[\frac{3}{2} + 4(\frac{7}{4}) + 2(\frac{3}{2}) + 4(\frac{5}{4}) + \frac{5}{2}\right] = \frac{1}{6}(19) = 3\frac{1}{6}$$

10. Determine whether each integral is convergent or divergent. Evaluate those that are convergent and identify those that are divergent.

(a) $\int_0^\infty xe^{-x^2}\,dx$

$\int_0^\infty xe^{-x^2}\,dx = \lim_{a\to\infty}\int_0^a xe^{-x^2}\,dx$

Let $u = -x^2$, then $du = -2x\,dx$, so we have

$= \lim_{a\to\infty}\int_{x=0}^{x=a}\frac{-1}{2}e^u\,du = \lim_{a\to\infty}\frac{-1}{2}(e^{-a^2}-1) = \frac{1}{2}.$

(b) $\int_{-1}^1 \frac{dx}{x^2}$

Divergent

(c) $\int_{-\infty}^\infty \frac{dx}{x^2+1}$

$\int_{-\infty}^\infty \frac{dx}{x^2+1} = \lim_{a\to-\infty}\int_a^0 \frac{dx}{x^2+1} + \lim_{b\to\infty}\frac{dx}{x^2+1}.$

Then $\lim_{a\to-\infty}\int_a^0 \frac{dx}{x^2+1} = \lim_{a\to-\infty}\arctan 0 - \arctan a = \frac{\pi}{2}$, and $\lim_{b\to\infty}\int_0^b \frac{dx}{x^2+1} = \lim_{b\to\infty}\arctan b - \arctan 0 = \frac{\pi}{2}$. Thus our answer is $\frac{\pi}{2}+\frac{\pi}{2}=\pi$.

11. Evaluate the following limits if they exist. If the limit does not exist, so state.

(a) $\lim_{n\to\infty}\frac{\ln n}{n}$

Use L'Hospital's rule to get $\lim_{n\to\infty}\frac{\ln n}{n} = \lim_{n\to\infty}\frac{1}{n} = 0$

(b) $\lim_{n\to\infty}\cos\frac{\pi}{n}$

Since cosine is a continuous function, we have $\lim_{n\to\infty}\cos\frac{\pi}{n} = \cos\left(\lim_{n\to\infty}\frac{\pi}{n}\right) = \cos 0 = 1$.

(c) $\lim_{n\to\infty}\left(1+\frac{\ln 3}{n}\right)^n$

Let $y = (1+\frac{\ln 3}{n})^n$. Then $\ln y = n\ln(1+\frac{\ln 3}{n})$, so $\lim_{n\to\infty} \ln y = \lim_{n\to\infty} \frac{1+\frac{\ln 3}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{1}{1+\frac{\ln 3}{n}}(\frac{-\ln 3}{n^2})}{\frac{-1}{n^2}} = \lim_{n\to\infty} \frac{\ln 3}{1+\frac{\ln 3}{n}} = \ln 3$. Thus our answer is $e^{\ln 3} = 3$.

Show your work for problems 12-15 (32%). Each problem is worth 8 points.

12. Find the centroid of the region between the curves $y = x^2$ and $y = 1$.

$A = \int_{-1}^{1} 1 - x^2\, dx = x - \frac{1}{3}x^3|_{-1}^{1} = \frac{4}{3}$.

$\overline{x} = \frac{3}{4}\int_{-1}^{1} x(1 - x^2)\, dx = 0$ since the integrand is an odd function.

$\overline{y} = \frac{3}{4}\int_{-1}^{1} \frac{1}{2}[(1)^2 - (x^2)^2]\, dx = \frac{3}{8}\int_{-1}^{1} \frac{(}{1} - x^4)\, dx = \frac{3}{8}(x - \frac{1}{5}x^5)_{-1}^{1} = \frac{3}{5}$. Thus the centroid is $(0, \frac{3}{5})$.

13. Evaluate the series $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+1)}$.

Notice that $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+1)} = 3\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$. Using partial fraction decomposition, we see that $\displaystyle\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$, so the sequence of partial sums is $\displaystyle s_n = 3\sum_{i=1}^{n}(\frac{1}{n} - \frac{1}{n+1}) = 3[(1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) + \cdots + (\frac{1}{n} - \frac{1}{n+1})] = 3(1 - \frac{1}{n+1})$. Thus we have $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+1)} = \lim_{n\to\infty} 3(1 - \frac{1}{n+1}) = 3$.

14. A region with area 4 lies in the first quadrant of the x-y plane. When the region is revolved about the x-axis, it sweeps out a volume of 20π. When revolved about the y-axis, it sweeps out a volume of 16π. Use the Theorem of Pappus to find the centroid of the region.

$V_1 = 20\pi = 2\pi\overline{y}(4)$, so $20\pi = 8\pi\overline{y} \Longrightarrow \overline{y} = \frac{5}{2}$.

$V_2 = 16\pi = 2\pi\overline{x}(4)$, so $16\pi = 8\pi\overline{x} \Longrightarrow \overline{x} = 2$.

Thus the centroid is $(2, \frac{5}{2})$.

15. Given a series $\sum_{i=1}^{\infty} a_i$.

(a) Define s_n, the nth partial sum.

$s_n = a_1 + a_2 + \cdots + a_n$

(b) Define what it means to write $\sum_{i=1}^{\infty} a_i = s$

Let $\{s_n\}$ be the sequence of partial sums as defined above. We say $\sum_{i=1}^{\infty} a_i = s$ if $\lim_{n\to\infty} s_n = s$ where s is a finite real number.